NEXT IAS

QUANTITATIVE APTITUDE

(Basic Numeracy & Data Interpretation)

Comprehensive Study Course

CIVIL SERVICES EXAMINATION 2025

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Quantitative Aptitude

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Contents

QUANTITATIVE APTITUDE

(Basic Numeracy & Data Interpretation)

UPSC SYLLABUS FOR CSAT

Total Marks : 200 **Duration : Two hours**

- Comprehension;
- Interpersonal skills including communication skills;
- Logical reasoning and analytical ability;
- Decision making and problem solving;
- General mental ability;
- Basic numeracy (numbers and their relations, orders of magnitude, etc.) (Class X level), Data interpretation (charts, graphs, tables, data sufficiency etc. — Class X level);

Paper-II of the Civil Services (Preliminary) Examination will be a qualifying paper with minimum qualifying marks fixed at 33%. The questions will be of multiple choice, objective type.

PREFACE

The journey to civil service examinations is one that is filled with dedication, perseverance, and relentless hard work. The Civil Services Aptitude Test (CSAT) is a crucial part of this journey, as it serves as the gateway to the prestigious Indian Civil Services. It is with great pleasure and immense pride that we present to you this book on "Quantative Aptitude" prepared by the NEXT IAS team under the guidance of **"Manjul Kumar Tiwari Sir"**.

The primary aim of this book is to provide aspirants with a thorough understanding of the CSAT examination pattern, the types of questions asked, and the best strategies to solve them. By providing detailed solutions to previous year questions, we hope to instill in you the confidence and ability to tackle any challenge that the CSAT may throw your way.

03 Chapter

Test of Divisibility & Indices

Test of Divisibility

A divisibility rule is a shorthand and useful way to determine whether a given number is divisible by a given divisor without actually performing the division, usually by examining its digits

Divisibility by 2

A number is divisible by 2 if the unit digit is zero or divisible by 2

Illus. 22, 42, 84 etc.

Divisibility by 3

A number is divisible by 3 if the sum of digits in the number is divisible by 3

Illus. In the number 5253,

Here, 5 + 2 + 5 + 3 = 15, which is divisible by 3 hence 5253 is divisible by 3

Divisibility by 4

A number is divisible by 4 if its last two digits are divisible by 4

Illus. 1652, here 52 is divisible by 4 so 1652 is divisible by 4.

Divisiblity by 5

A number is divisible by 5 if the unit's digits in number is 0 or 5

Illus. 50, 505, 555 etc.

Divisibility by 6

A number is divisible by 6 if the number is even and sum of digits is divisible by 3

Illus. 5346 is an even number, also sum of digit

5 + 3 + 4 + 6 = 18 is divisible by 3

Divisibility by 8

A number is divisible by 8 if last three digits of it is divisible by 8

Illus. 47472 here 472 is divisible by 8 hence this number 47472 is divisible by 8

Divisibility by 9

A number is divisible by 9 if sum of its digits is divisible by 9

Illus. 108936 here 1 + 0 + 8 + 9 + 3 + 6 is 27 which is divisible by 9 and hence 108936 is divisible by 9

Divisibility by 10

A number is divisible by 10 if its unit digit is 0

Illus. 90, 900, 740 etc.

Divisibility by 11

A number is divisible by 11 if the difference of sum of digit at odd places and sum of digit at even places is either 0 or divisible by 11

Illus. 1331, the sum of digits at odd place is 1 + 3 and sum of digit at even places is 3 + 1 and their difference is 4 – 4 = 0. So 1331 is divisible by 11.

Ex. How many distinct values can x assume if $17327x4$ is divisible by 8?

(a) 0 (b) 1

(c) 2 (d) More than 2

Sol. **(d)**

x can take value from 0, 1, 2, ... 9.

To be divisible by 8 i.e., last 3 digits must be divisible by 8. i.e., $7x4$ must be divisible by 8.

Putting x = 0, 1, ...9 and checking,

For x = 0, 4, 8; number is divisible by 8.

Ex. If the number $31285x57y$ is divisible by 72, find $x + y$

Sol. Number is divisible by 72

$\Rightarrow$ It should be divisible by both 8 and 9

First we will check the divisibility of 8

$\Rightarrow$ $57y$ must be divisible by 8

$\Rightarrow$ y = 6, so last three digit are 576

Now number becomes 31285x576 which is divisible by 9 so,

$$\text{Rem}\left(\frac{3+1+2+8+5+x+5+7+6}{9}\right)=0$$

$$\Rightarrow \text{Rem}\left(\frac{37+x}{9}\right)=0$$

$\Rightarrow x = 8$

So, $x + y = 6 + 8 = 14$

Cyclicity of Numbers

- Cyclicity of any number is about the last or unit's digit and how they appear in a certain defined manner
- Cyclicity of a number is used mainly for the calculation of unit digits

Cyclicity of 1

In 1^n, unit digit will always be 1

Cyclicity of 2

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

After every four intervals it repeats so cycle of 2 is 2, 4, 8, 6

Illus. Find the unit digit of 2^{332}

Here 2, 4, 8, 6 will repeat after every four interval till power 320, next digit will be 2, $\boxed{4}$. So unit digit of 2^{322} will be 4

Cyclicity of 3

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

$3^6 = 729$

$3^7 = 2187$

$3^8 = 6561$

After every four intervals 3, 9, 7 and 1 are repeated. So cycle of 3 is 3, 9, 7, 1.

Illus. Find unit digit of 33^{133}

Sol. Cycle of 3 is 3, 9, 7, 1 which repeats after every four intervals till 33^{132}. So, next unit digit will be 3

Cyclicity of 4

$4^1 = 4$

$4^2 = 16$

$4^3 = 64$

$4^4 = 256$

Cycle is 4, 6, i.e.

Unit digit of 4^n depends on value of n

If n is odd unit digit is 4 and if n is even digit is 6

Illus. Find unit digit of 4^{52}

Sol. Since 52 is even number unit digit will be 6

Cyclicity of 5

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$5^4 = 625$

Unit digit will always be 5

Cyclicity of 6

$6^1 = 6$

$6^2 = 36$

$6^3 = 216$

$6^4 = 1296$

Unit digit will always be 6

Illus. Find unit digit of $4^{29} \times 6^7$

Sol. Unit digit of 4^{29} is 4 and unit digit of 6^7 is 6 so unit digit of $4^{29} \times 6^5$ will be $4 \times 6 = 24$ i.e. 4

Cyclicity of 7

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$

$7^5 = 16807$

$7^6 = 117649$

$7^7 = 823543$

$7^8 = 5764801$

Cycle of 7 is 7, 9, 3, 1

Illus. Find unit digit of 17^{17}

Sol. Unit digit of 17^{17} is 7

Cyclicity of 8

$8^1 = 8$

$8^2 = 64$

$8^3 = 512$

$8^4 = 4096$

$8^5 = 32768$

So, cycle of 8 is 8, 4, 2, 6

Illus. Find unit digit of $18^{18} \times 28^{28}$

Sol. Unit digit of 18^{18} is 4 and unit digit of 28^{28} is 6.

So unit digit of $18^{18} \times 28^{28}$ will be $4 \times 6 = 24$ i.e., 4

Cyclicity of 9

$9^1 = 9$

$9^2 = 81$

$9^3 = 729$

$9^4 = 6561$

Cycle of 9 is 9, 1

In 9^n unit digit will be 9 if n is odd and unit digit will be 1 if n is even

Ex. Find unit digit

$10^{10} + 11^{11} + 12^{12} + 13^{13} + 14^{14} + 15^{15}$

Sol. Unit digit of 10^{10} is 0

Unit digit of 11^{11} is 1

Unit digit of 12^{12} is 6

Unit digit of 13^{13} is 3

Unit digit of 14^{14} is 6

Unit digit of 15^{15} is 5

So unit digit of given sum will be $0 + 1 + 6 + 3 + 6 + 5 = 21$ i.e., 1

Ex. The units digit of $(2017)^{2017}$ is

(a) 3 (b) 7
(c) 1 (d) 9

Sol. **(b)**

The last digit of 2017 is 7 and the cyclicity of 7 is four. So, the last digit of $(2017)^{2017}$ will be the Ist term in the cycle of 7^n i.e., 7

Ex. The unit digit of $(216)^{216} + (217)^{217}$

(a) 3 (b) 7
(c) 5 (d) 9

Sol. **(a)**

The last digit of 216 is 6 and the cyclicity of 6 is one hence, last digit of $(216)^{216}$ will be 6

Similarly, the last digit of 217 is 7 and the cyclicity of 7 is four. As $\frac{217}{4}$ gives the remainder 1, so the last digit of $(217)^{217}$ will be 7

$\therefore$ Last digit of $(216)^{216} + (217)^{217}$ = Last digit of $(6 + 7) = 3$

Ex. The unit digit of the number $(4)^{400} \times (9)^{900}$ is

(a) 3 (b) 6
(c) 7 (d) 1

Sol. **(b)**

Since power is even, unit digit of $4^{400} = 6$

Similarly, as the cyclicity of 9 is two, and $\frac{900}{2}$ gives the zero remainder. So, again the last digit of 9^{900} will be the last term in the cycle of 9^n i.e., 1. Hence, unit digit of the product $4^{400} \times 9^{900}$ = Unit digit of $(6 \times 1) = 6$

Ex. The rightmost non-zero digit of the number $(40)^{2100}$ is

(a) 1 (b) 3
(c) 7 (d) 9

Sol. **(a)**

As $(40)^{2100}$ can be written as $4^{2100} \times 10^{1200}$ so, the rightmost non-zero digit of 4^{2100} = Rightmost non-zero digit of $(4)^{2100}$

The cyclicity of 4 is two. As $\frac{2100}{2}$ gives zero remainder so, the last digit of 4^{2100} will be 6

Remainder Theorem

Remainder of expression $\frac{a \times b \times c}{n}$ is equal to the remainder of expression $\frac{a_n \times b_n \times c_n}{n}$,

where, a_n is remainder when a is divided by n

b_n is remainder when b is divided by n, and

c_n is remainder when c is divided by n

Illus. Find the remainder of 13 × 17 × 19 when it is divided by 7

Sol. Remainder of $\frac{13}{7} = 6$

Remainder of $\frac{17}{7} = 3$

Remainder of $\frac{19}{7} = 5$

$\therefore$ Remainder of $\frac{13 \times 17 \times 19}{7}$

= Remainder of $\left(\frac{6 \times 3 \times 5}{7}\right)$ = Remainder $\frac{90}{7} = 6$

Ex. Find the remainder when 5^5 is divided by 3

(a) 0 (b) 1

(c) 2 (d) None of these

Sol. **(c)**

$$\frac{5^5}{3} = \frac{5 \times 5 \times 5 \times 5 \times 5}{3}$$

Using remainder theorem,

$$\text{Rem}\left(\frac{5^5}{3}\right) = \text{Rem}\left(\frac{2 \times 2 \times 2 \times 2 \times 2}{3}\right) = -\left(\frac{32}{3}\right) = 2$$

Ex. Find the remainder when 2^{72} is divided by 7

(a) 1 (b) 3

(c) 5 (d) 6

Sol. **(a)**

$2^{72} = (2^3)^{24} = (8)^{24}$

$\therefore$ Remainder $\left(\frac{8^{24}}{7}\right) = \text{Rem}\left(\frac{1^{24}}{7}\right) = 1$

Ex. Find the remainder when (237 × 2318 – 1235 + 4127) is divided by 4

Sol. $\text{Rem}\left(\frac{237 \times 2318 - 1234 + 412}{4}\right)$

$= \text{Rem}\left(\frac{237}{4}\right) \times \text{Rem}\left(\frac{2318}{4}\right) - \text{Rem}\left(\frac{1235}{4}\right)$

$+ \text{Rem}\left(\frac{4127}{4}\right)$

= 1 × 2 – 3 + 3 = 2

Powers / Indices

- It is used to show or write large expressions
- When a quantity repeatedly multiplies with itself
- $a \times a \times a \times a \ldots m$ times $= a^m$
- Here a^m is called m^{th} power of a

Illus. $2 \times 2 \times 2 \times 2 = 16 = 2^4$

a^m — m: power/index, a: base

Laws/Rules of Indices:

1. $\underbrace{a^m \times a^n}_{\text{Base same}} = a^{\overbrace{m+n}^{\text{Powers add}}}$

Illus. $2^3 \times 2^4 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 2^{(3+4)}$

2. $\frac{a^m}{a^n} = a^{m-n}$

Illus. $\frac{2^6}{2^4} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = 2^2 = 2^{(6-4)}$

3. $(a^m)^n = a^{m \times n} = (a^n)^m = (a^m)^n$

Illus. $(2^4)^3 = (2 \times 2 \times 2 \times 2)^3 = 2 \times 2 \ldots 12$ times

$= 2^{12} = 2^{3\times4}$

$= (2^3)^4 = 2^{12}$

4. $(ab)^m = a^m \times b^m$

Illus. $(2 \times 3)^4 = (2 \times 3 \times 2 \times 3 \times 2 \times 3 \times 2 \times 3)$

$= (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$

$= 2^4 \times 3^4$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Illus. $\left(\frac{2}{3}\right)^5 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2^5}{3^5}$

6. $a^0 = 1$ (a may be any number)

Note: 0^0 is undefined

7. $a^{-m} = \frac{1}{a^m}$

8. $a^m = a^n$

If base is same then power is same *i.e.,* $m = n$

Illus. Find x if $\left(\frac{a}{b}\right)^{2x-4} = \left(\frac{b}{a}\right)^{4x-3}$

Sol. $\left(\frac{a}{b}\right)^{2x-4} = \left(\frac{a}{b}\right)^{-4x+3}$

$\Rightarrow \quad 2x - 4 = 4x - 3$

$\Rightarrow \quad -1 = 2x$

$\Rightarrow \quad x = -\frac{1}{2}$

Ex. If $4^x 2^y = 128$ and $3^{3x} \times 3^{2y} = 9^{xy}$, find $x + y$

Sol. $2^{2x+y} = 2^7$ and $3^{3x+2y} = 3^{2xy}$

$\Rightarrow \quad 2x + y = 7$ and $3x + 2y = 2xy$

$\Rightarrow \quad x + y + 7 = 2xy$

$\Rightarrow \quad x + 7 + (7 - 2x) = 2x(7 - 2x)$

$\Rightarrow \quad 14 - x = 14x - 4x^2$

$\Rightarrow \quad 4x^2 - 15x + 14 = 0$

$4x^2 - 8x - 7x + 14 = 0$

$(4x - 7)(x - 2) = 0$

$\Rightarrow \quad x = 2, \frac{7}{4}$

$y = 3, \frac{7}{2}$

$\therefore x + y = 5, \frac{21}{4}$

Ex. Solve: $\frac{(a+b)^4 \times (a+b)^{\frac{-3}{6}}}{\sqrt{(a+b)}}$

Sol. $\frac{(a+b)^4 \times (a+b)^{\frac{-1}{2}}}{(a+b)^{\frac{1}{2}}}$

$= \frac{(a+b)^{4-\frac{1}{2}}}{(a+b)^{\frac{1}{2}}} = (a+b)^{\frac{7}{2}-\frac{1}{2}} = (a+b)^3$

Ex. Solve the following expression:

$$\frac{(a+b)^3 \div (a+b) \div (a+b)}{\left[2(a+b)^{\frac{7}{2}} - (a+b)^{\frac{7}{2}}\right] \times (a+b)^{\frac{1}{2}}}$$

Sol. $$\frac{(a+b)^3 \times \frac{1}{(a+b)^2} \times (a+b)}{(a+b)^{\frac{7}{2}} \times (a+b)^{\frac{1}{2}}}$$

$$\frac{(a+b)^{3-2+1}}{(a+b)^{\frac{7}{2}+\frac{1}{2}}} = \frac{(a+b)^2}{(a+b)^4} = (a+b)^{-2}$$

Surds

Surds are powers in fractions (i.e., when power < 1)

Illus. $a^2 = a \times a$

Let $a^2 = b$

Then, $a = b^{\frac{1}{2}} = \sqrt{b}$

- $\sqrt{b}$ – square root of b
- Similarly, $b^{\frac{1}{3}}$ – cube root of b
- $b^{\frac{1}{n}}$ or $\sqrt[n]{b}$ – surd of order 'n'
- (By default '2') $\longrightarrow \sqrt{b}$

Laws/Rules of Surds:

Rules of Surds are similar to that of indices, we can simply replace m by $\frac{1}{n}$

1. $a^b \times a^c = a^{b+c}$

$a^{1/b} \times a^{1/c} = a^{1/b+1/c}$

Illus. $2^{1/2} \times 2^{1/3} = 2^{5/6}$

2. $\left(\sqrt[n]{a}\right)^n = \left(a^{1/n}\right)^n = a^{n/n} = a$

3. $\left(\frac{a}{b}\right)^{1/n} = \frac{a^{1/n}}{b^{1/n}}$

4. $(a^{1/n})^m = a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$

5. $\sqrt[n]{\left(\sqrt[m]{a}\right)} = \sqrt[n]{\left(a^{\frac{1}{m}}\right)} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{m}\times\frac{1}{n}} = a^{\frac{1}{mn}}$

$= \sqrt[mn]{a} = \left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = \sqrt[m]{\left(\sqrt[n]{a}\right)}$

6. $(ab)^{\frac{1}{m}} = (a)^{\frac{1}{m}}(b)^{\frac{1}{m}}$

Ex. Solve $(64)^{-\frac{2}{3}} \times \left(\frac{1}{4}\right)^{-2}$.

Sol. $\frac{1}{(64)^{2/3}} \times 4^2 = \frac{16}{\left(\sqrt[3]{64}\right)^2} = \frac{16}{16} = 1$

Ex. If $\left(\sqrt{3}\right)^5 \times 9^2 = 3^k \times 3\sqrt{3}$, then $k = ?$

Sol. $3^{5/2} \times 3^4 = 3^k \times 3^{1+\frac{1}{2}}$

$\frac{5}{2} + 4 = k + 1 + \frac{1}{2}$

$k = 5$

Ex. Arrange the following in ascending/descending order.

$2^{1/2}, 3^{1/3}, 4^{1/4}, 6^{1/6}, 12^{1/12}$

Sol. If a > b

$\Rightarrow a^m > b^m$ for a, b, m positive

Method to solve such questions:

Take LCM of denominators of powers and raise the powers by it i.e. change powers from fraction to whole numbers.

LCM (2, 3, 4, 6, 12) = 12

$\Rightarrow 2^{1/2}, 3^{1/3}, 4^{1/4}, 6^{1/6}, 12^{1/12} = 2^6, 3^4, 4^3, 6^2, 12^1$

$\Rightarrow$ 64, 81, 64, 36, 12

$\therefore$ Largest number = 81

i.e., largest number = $3^{\frac{1}{3}}$

Ex. Compare $\sqrt{7}-\sqrt{5}, \sqrt{5}-\sqrt{3}, \sqrt{9}-\sqrt{7}, \sqrt{11}-\sqrt{9}$ to find the greatest number

Sol. $\sqrt{7}-\sqrt{5} = \sqrt{7}-\sqrt{5} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{7-5}{\sqrt{7}+\sqrt{5}}$

$= \frac{2}{\sqrt{7}+\sqrt{5}}$

Similarly,

$\frac{2}{\sqrt{7}+\sqrt{5}}, \frac{2}{\sqrt{5}+\sqrt{3}}, \frac{2}{\sqrt{9}+\sqrt{7}}, \frac{2}{\sqrt{11}+\sqrt{9}}$

Smallest denominator = greatest fraction

Therefore, greatest number = $\frac{2}{\sqrt{5}+\sqrt{3}}$

Ex. Arrange the following:

$12-2\sqrt{35}, 8-2\sqrt{15}, 16-2\sqrt{63}, 20-2\sqrt{99}$

Sol. $12-2\sqrt{35} = 7+5 -2\sqrt{7}\times\sqrt{5} = (\sqrt{7}-\sqrt{5})^2$

Similarly, $8-2\sqrt{15} = (\sqrt{5}-\sqrt{3})^2$

$16-2\sqrt{63} = (\sqrt{9}-\sqrt{7})^2$

$20-2\sqrt{99} = (\sqrt{11}-\sqrt{9})^2$

Now, $(\sqrt{7}-\sqrt{5})^2 = \left(\frac{2}{\sqrt{7}+\sqrt{5}}\right)^2$

Similarly, $(\sqrt{5}-\sqrt{3})^2 = \left(\frac{2}{\sqrt{5}+\sqrt{3}}\right)^2$

$(\sqrt{9}-\sqrt{7})^2 = \left(\frac{2}{\sqrt{9}+\sqrt{7}}\right)^2$

$(\sqrt{11}-\sqrt{9})^2 = \left(\frac{2}{\sqrt{11}+\sqrt{9}}\right)^2$

Greatest denominator = Smallest fraction

$\therefore$ in ascending order,

$20-2\sqrt{99}, 16-2\sqrt{63}\ 12-2\sqrt{35}, 8-2\sqrt{15}$

Square Root

The square root of a number is that number, the square of which is equal to the given number. There are two types of square roots of a number, positive and negative. It is denoted by the sign '$\sqrt{\ }$'

Illus. 64 has two square roots 8 and –8, because $(8)^2 = 64$ and $(-8)^2 = 64$. Hence, we (can write $\sqrt{64} = \pm 8$).

Methods to Find Square Root

There are primarily 2 methods to find square root of a number.

Method I: Prime Factorisation Method

Step 1: Express the given number as the product of prime factors.

Step 2: Arrange the factors in pairs of same prime numbers.

Step 3: Take the product of these prime factors taking one out of every pair of the same primes. This product gives us the square root of the given number.

Illus. Find the square root of 1156.

Sol. Prime factors of 1156 = 17 × 17 × 2 × 2

$\Rightarrow$ 1156 = 17 × 17 × 2 × 2

Now, taking one number from each pair and multiplying them, we get

$\sqrt{1156} = 17 \times 2 = 34$

Method II: Division Method

Understand with the help of following illustration,

Illus. Find the square root of 3249.

Sol. **Step 1:** In the given number, mark off the digits in pairs starting from the unit digit. Each pair and the remaining one-digit is called a period.

Step 2: Now, $5^2 = 25$; On subtracting 25 from 32 we get (7) as remainder.

Step 3: Bring down the next period i.e. 49 Now, the trial divisor is $5 \times 2 = 10$ and trial dividend is 749. So, we take 107 as divisor and put 7 as quotient. The remaidner is 8 now.

Step 4: This process (Step II and III) goes on till all the periods (pairs) come to an end and we get remainder as 0 (zero) now

	57
	3249
5	−25
107	749
	−749
	0

Hence, the required square root = 57

Ex. What is the square root of 12769 ?

Sol. ∴ Required square root = 113

	113
	12769
1	−1
21	027
	−21
223	669
	−669
	0

Square Root of a Fraction

To find square root of a fraction, we have to find the square roots of numerators and denominators, separately.

Ex. Find $\sqrt{\dfrac{2809}{64}}$

Sol. $\sqrt{\dfrac{2809}{64}} = \dfrac{\sqrt{2809}}{\sqrt{64}} = \dfrac{53}{8}$

Ex. Find the square root of $\dfrac{2450}{32}$

Sol. $\sqrt{\dfrac{2450}{32}} = \sqrt{\dfrac{1225}{16}} = \dfrac{\sqrt{1225}}{\sqrt{16}} = \dfrac{35}{4}$

Ex. Find the square of $\sqrt{5\sqrt{5\sqrt{5\sqrt{.......\infty}}}}$

Sol. Let $x = \sqrt{5\sqrt{5\sqrt{5\sqrt{.......\infty}}}}$

$\therefore\ x^2 = 5\sqrt{5\sqrt{5\sqrt{5...\infty}}} = 5x$

$\Rightarrow\ x^2 = 5x$

$\Rightarrow\ x = 5$

Ex. Find the square of $\sqrt{42+\sqrt{42+\sqrt{42+...}}}$

Sol. Let $x = \sqrt{42+\sqrt{42+\sqrt{42+...\infty}}}$

$\Rightarrow\ x^2 = 42+\sqrt{42\sqrt{42+\sqrt{42}}} = \infty = 42 + x$

$\Rightarrow\ x^2 - x - 42 = 0$

$\Rightarrow\ (x-7)(x+6) = 0$

$\Rightarrow\ x = 7$ (x can't be negative)

Cube Root

The cube root of a given number is the number whose cube is the given number. The cube root is denoted by the sign $\sqrt[3]{\ }$

Illus. (i) $\sqrt[3]{8} = \sqrt[3]{2\times2\times2} = 2$

(ii) $\sqrt[3]{512} = \sqrt[3]{8\times8\times8\times} = 8$

Method to find cube root of a given number:

Prime Factorisation Method

Step 1: Express the given number as the product of prime factors

Step 2: Arrange the factors in a group of three of same prime numbers

Step 3: Take the product of these prime factors picking one out of every group (group of three) of the same primes. This product gives us the cube root of given number

Illus. Find the cube root of 10648

Sol. Prime factors of 10648 = $(2 \times 2 \times 2) \times (11 \times 11 \times 11)$

$\Rightarrow \sqrt[3]{10648} = \sqrt[3]{2\times2\times2\times11\times11\times11}$

Now, taking one number from each group of three, we get

$\sqrt[3]{10648} = 2 \times 11 = 22$

Ex. Find the value of $\sqrt[3]{\frac{0.000729}{0.032768}}$

Sol. $\sqrt[3]{\frac{0.000729}{0.032768}} = \sqrt[3]{\frac{729}{32768}} = \sqrt[3]{\frac{9\times9\times9}{32\times32\times32}} = \frac{9}{32}$

Ex. Find the cube root of –5832

Sol. $\sqrt[3]{(-5832)} = -\sqrt[3]{5832} = -\sqrt[3]{18\times18\times18} = -18$

Logarithms

- Logarithm is used to answer how many times base must be multiplied by itself to get another number.

- $a^x = c$ can be written as $\log_a^c = x$
  where, a – base
  x – Power/ index
  c – number

Illus. $2^3 = 8$

$\log_2 8 = 3$

Note:

- $\log_a a = 1$
- $\log_a 1 = 0$

Laws/ Rules of Logarithms

1. $\underbrace{\log_a(m\times n)}_{c} = \underbrace{\log_a m}_{x} + \underbrace{\log_a n}_{y}$

$\Rightarrow \quad m \times n = a^c$

also, $\quad m = a^x$

$n = a^y$

$\Rightarrow \quad m \times n = a^{x+y}$

$\Rightarrow \quad x + y = c$

2. $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

3. $\log_a(m^n) = n\log_a m$

Illus. $\log_a(m \times m ...) = \log_a m + \log_a m ... = n\log_a m$

5. $\log_a b = \frac{\log_n b}{\log_n a}$

6. $\log_a b = \frac{\log_n b}{\log_n a} = \frac{1}{\frac{\log_n a}{\log_n b}} = \frac{1}{\log_b a}$

Points to Remember:

- By default base is 10
- When base = 10, it is called common log (log)
- When base = e, it is called natural log (ln)

Ex. Find $a^{\log_a n}$

Sol. $a^{\log_a n} = k$

$\log_a n \log a = \log k$

$\Rightarrow \frac{\log n}{\log a}\log a = \log k$

$\Rightarrow k = n$

Ex. If $x = y^a$, $y = z^b$, $z = x^c$, $abc = ?$

Sol. $\log x = a\log y$

$\log y = b\log z$

$\log z = c\log x$

$abc = \frac{\log x}{\log y}\times\frac{\log y}{\log z}\times\frac{\log z}{\log x} = 1$

Solved Examples

Q.1 The numbers 1 to 18 are written side by side as follows 1234567891011 ... 1718. If the number is divided by 9, then what is the remainder?

(a) 0 (b) 4
(c) 3 (d) 7

Sol. **(a)**

Sum of digits from 1 to 9 = 45

Sum of digits from 10 to 18 = 9 + 36 = 45

Sum of the digits from 1 to 18 = 45 + 45 = 90

Since, sum is divisible by 9

∴, Remainder = 0

Q.2 If x and y are the two digits of the number 213 xy such that this number is divisible by 60, then maximum of $x + y$ is equal to :

(a) 2 (b) 4
(c) 6 (d) 8

Sol. **(c)**

Since 213xy is divisible by 5 as well as 2, so $y = 0$

Now, 213 x0 must be divisible by 12

So, 213x0 must be divisible by 4 and 3

For divisibility by 4

x = 0, 2, 4, 6, 8

For divisibility by 3,

2 + 1 + 3 + x + 0 = 6 + x must be divisible by 3

$\Rightarrow x = 0$ or 6

For $x + y$ as maximum,

x must be 6

$\Rightarrow x + y = 6$

Q.3 $10^{10} + 11^{11} + 12^{12} + 13^{13} + 14^{14} + 15^{15}$ unit digit of 10^{10} is 0

Sol. Unit digit of 11^{11} is 1

Unit digit of 12^{12} is 6

Unit digit of 13^{13} is 3

Unit digit of 14^{14} is 6

Unit digit of 15^{11} is 1

So unit digit of given sum will be

0 + 1 + 6 + 3 + 6 + 5 = 21 i.e. 1

Q.4 Find the remainder of $\frac{9^{99}}{8}$

Sol. $\frac{9^{99}}{8} = \frac{(8+1)^{99}}{8}$

According to Remainder theorem, remainder will be equal to remainder of the expression $\frac{1^{99}}{8}$ which is equal to 1

Q.5 Find remainder of $\frac{5^{100}}{7}$

(a) 1 (b) 2

(c) 4 (d) None of these

Sol. **(b)**

$$\frac{5^{100}}{7} = \frac{(5^2)^{50}}{7} = \left[\frac{3\times7+4}{7}\right]^{50} \Rightarrow \text{Rem}\left[\frac{(4)^{50}}{7}\right]$$

$$\Rightarrow \text{Rem}\left[\frac{2^{100}}{7}\right] = \frac{(2^3)^{33}\times2}{7} \Rightarrow \frac{(7+1)^{33}}{7}\times2$$

$$\Rightarrow \frac{1\times2}{7}$$

$\Rightarrow$ Remainder is 2

Q.6 If $X381$ is divisible by 11, find the value of the smallest natural number X?

(a) 3 (b) 4

(c) 7 (d) 9

Sol. **(c)**

$X381$ is divisible by 11 if $(X + 8) - (3 + 1)$ is divisible by 11

So, $X = 7$ satisfies the condition

Q.7 Fill in the blank in the number 4 _ 56 so as to make it divisible by 33

(a) 3 (b) 4

(c) 5 (d) Data Insufficient

Sol. **(a)**

4 _ 56 is divisible by 33 if and only if it is divisible by 3 and 11

4 _ 56 will be divisible by 3 if _ will be equal to 0, 3, 6, 9

4 _ 56 is divisible by 11 if (4 + 5) – (_ + 6) will be divisible by 11, so _ should be 3

Q.8 Check if the expression $333^{555} + 555^{333}$ is divisible by 2

Sol. Check for divisibility by 2

$$\text{Rem}\left\{\frac{(333)^{555} + (555)^{333}}{2}\right\}$$

$$\Rightarrow \text{Rem}\left(\frac{1^{555} + 1^{333}}{2}\right)$$

$\Rightarrow$ Rem = 0

$\Rightarrow$ Divisible by 2

Q.9 Find the remainder of 2^{100} when divided by 3

(a) 2 (b) 0

(c) 1 (d) None of these

Sol. **(c)**

$$\text{Remainder of }\left(\frac{2^{100}}{3}\right) = R\left\{\frac{(2^{10})^{10}}{3}\right\}$$

$$\Rightarrow R\left\{\frac{(1024)^{10}}{3}\right\} \Rightarrow R\left(\frac{1024.1024...10\text{ times}}{3}\right)$$

$$\Rightarrow R\left(\frac{1.1.1...10\text{ times}}{3}\right) \Rightarrow \text{Remainder} = 1$$

So, option 'c' is correct.

Q.10 Find the units digit of $3^{69} \times 6^{59} + 7^{75}$

Sol. Unit digit = 3 × 6 + 3 = 8 + 3 = 1

Q.11 What is the remainder when 241 × 2310 – 1243 + 4127 is divided by 4 ?

Sol. Using Remainder Theorem,

Remainder $\left(\frac{241\times2310-1243+41272}{4}\right)$

$= \text{Remainder}\left(\frac{1\times2-3+3}{4}\right) = 2$

Q.12 Find the remainder when $241^3 \times 2314^2 - 123^5 + 412$ is divided by 11

Sol. Using Remainder Theorem,

Remainder $\left(\frac{241^3\times2314^2-123^5+412}{11}\right)$

$= \text{Remainder}\left(\frac{10\times10\times10\times4\times4-2^5+5}{11}\right)$

$= \text{Remainder}\left(\frac{6-10+5}{11}\right) = 1$

Q.13 Find the remainder of $\frac{4^{33}}{7}$

Sol. $4^{33} = (4^3)^{11} = 64^{11}$

Remainder $\left(\frac{64^{11}}{7}\right) = \text{Remainder}\left(\frac{1^{11}}{7}\right) = 1$

Q.14 Find smallest number among the following

$\frac{1}{\sqrt{3}-1}, \frac{1}{\sqrt{7}-\sqrt{5}}, \frac{1}{\sqrt{9}-\sqrt{7}}, \frac{1}{\sqrt{13}-\sqrt{11}}$

Sol. $\frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2}$

Similarly, $\frac{\sqrt{7}+\sqrt{5}}{2}, \frac{\sqrt{9}+\sqrt{7}}{2}, \frac{\sqrt{13}+\sqrt{11}}{2}$

Therefore, smaller number $= \frac{1}{\sqrt{3}-1}$

Q.15 Find the largest number among

$2\sqrt{3}, 2\sqrt[4]{5}, \sqrt{8}, 3\sqrt{2}$

Sol. LCM (2, 4) = 4

$\Rightarrow \left[2(\sqrt{3})\right]^4, \left[2(\sqrt[4]{5})\right]^4, (\sqrt{8})^4, (3\sqrt{2})^4$

$\Rightarrow$ 144, 80, 64, 324

$\therefore$ Largest number = $3\sqrt{2}$

Q.16 Find the value of $\sqrt{5+2\sqrt{6}} - \frac{1}{\sqrt{5+2\sqrt{6}}}$

Sol. $5+2\sqrt{6} = (\sqrt{2}+\sqrt{3})^2$

$\Rightarrow \sqrt{(\sqrt{2}+\sqrt{3})^2} - \frac{1}{\sqrt{(\sqrt{2}+\sqrt{3})^2}}$

$= \sqrt{2}+\sqrt{3} - \frac{1}{\sqrt{3}+\sqrt{2}}$

$= \sqrt{3}+\sqrt{2} - \frac{(\sqrt{3}-\sqrt{2})}{1} = 2\sqrt{2}$

Q.17 Rationalise $\frac{2}{\sqrt{3}}$

Sol. Rationalising means no root in denominator

$\Rightarrow \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Q.18 Arrange $\frac{1}{\sqrt[3]{12}}, \frac{1}{\sqrt[4]{29}}, \frac{1}{\sqrt{5}}$ in ascending order

Sol. LCM (3, 4, 2) = 12

$\Rightarrow \frac{1}{29^3}, \frac{1}{12^4}, \frac{1}{5^6}$

Q.19 Find square root of $5+\sqrt{6}$

Sol. Let, $\sqrt{5+\sqrt{6}} = \sqrt{x}+\sqrt{y}$

$\left(\sqrt{5+\sqrt{6}}\right)^2 = \left(\sqrt{x}+\sqrt{y}\right)^2$

$\Rightarrow 5+\sqrt{6} = x+y+2\sqrt{xy}$

Comparing both sides,

$5 = x + y$ ...(i)

and, $\sqrt{6} = 2\sqrt{xy}$

$xy = \frac{3}{2}$ ...(ii)

$(x - y)^2 = (x + y)^2 - 4xy = 25 - 6 = 19$

$\Rightarrow x - y = \sqrt{19}$

$x + y = 5$

$\therefore x = \frac{5+\sqrt{19}}{2}, y = \frac{5-\sqrt{19}}{2}$

Q.20 If $\sqrt{x}+\frac{1}{\sqrt{x}} = \sqrt{6}$, find $x^2+\frac{1}{x^2}$

(a) 14 (b) 12
(c) $2\sqrt{2}$ (d) None of these

Sol. (a)

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = (\sqrt{6})^2$$

$$\Rightarrow x+\frac{1}{x}+2 = 6$$

$$\Rightarrow x+\frac{1}{x} = 4$$

$$\Rightarrow x^2+\frac{1}{x^2}+2 = 16$$

$$\therefore x^2+\frac{1}{x^2} = 14$$

Q.21 What is 'x' if $81\times\left(\frac{16}{25}\right)^{x+2}\div\left(\frac{3}{5}\right)^{2x+4} = 144$

(a) –3 (b) 1
(c) –1 (d) 3

Sol. (c)

$$81\times\left(\frac{4}{5}\right)^{2(x+2)}\times\left(\frac{5}{3}\right)^{2x+4}$$

$$= 3^4\times\frac{4^{2x+4}}{5^{2x+4}}\times\frac{5^{2x+4}}{3^{2x+4}} = \frac{4^{2x+4}}{3^{2x}} = \frac{2^{4x+8}}{3^{2x}} = 144$$

$$\Rightarrow \frac{2^{4x+8}}{3^{2x}} = 2^4\cdot 3^2$$

$$\Rightarrow 2^{4x+8}\cdot 3^{-2x} = 2^4\cdot 3^2$$

$\Rightarrow 4x + 8 = 4\ ;\ -2x = 2$

$\Rightarrow x = -1$

Q.22 If $\underbrace{a+a+a+\ldots}_{n\text{ times}} = a^2b$ and $\underbrace{b+b+b\ldots}_{m\text{ times}} = ab^2$ then what is the value of $\left[\underbrace{m+m\ldots}_{n\text{ times}}\right]\left[\underbrace{n+n\ldots}_{m\text{ times}}\right]$

(a) ab (b) $(ab)^3$
(c) $(ab)^2$ (d) $(ab)^4$

Sol. (d)

$na = a^2b$

$\Rightarrow n = ab$

Also, $mb = ab^2$

$\Rightarrow m = ab$

$mn \times nm = (mn)^2 = (ab)^4$

Q.23 If $\log_4 x + \log_2 x = 6$, then x = ?

Sol. $\log_{2^2} x = \frac{1}{2}\log_2 x$

$$\Rightarrow \frac{3}{2}\log_2 x = 6$$

$\therefore \log_2 x = 4 \Rightarrow x = 16$

Q.24 $\frac{\log P}{y-z} = \frac{\log Q}{z-x} = \frac{\log R}{x-y} = 10$, PQR = ?

Sol. $P = 10^{10(y-z)}$

$Q = 10^{10(z-x)}$

$R = 10^{10(x-y)}$

$PQR = 10^{10(x-y+y-z+z-x)} = 10^0 = 1$

Q.25 If $\log P = \frac{1}{2}\log Q = \frac{1}{3}\log R$ then which of the following is true ?

(a) $P^2 = Q^2R^2$ (b) $Q^2 = PR$
(c) $Q^2 = R^3P$ (d) $R = P^2Q^2$

Sol. (b)

Let, $\log P = k$ $\Rightarrow P = e^k$

then $\log Q = 2k$ $\Rightarrow Q = e^{2k} = P^2$

and $\log R = 3k$ $\Rightarrow R = e^3k = P^3$

$\therefore Q^2 = P^4 = P \times P^3 = PR$

Previous Years Solved Questions

Q.1 How many numbers from 0 to 999 are not divisible by either 5 or 7?

(a) 313 (b) 341
(c) 686 (d) 786

Sol. (c)

Numbers from (0 – 999), divisible by 7

$$= \frac{999}{7} = 142\frac{5}{7} \approx 143$$

Numbers from to (0 - 999) divisible by 5

$$= \frac{999}{5} = 199\frac{4}{5} \approx 200$$

There are few number which are divisible by both 5 and 7 i.e. by 35

Numbers from (0 – 999) divisible by 35

$$= \frac{999}{35} \approx 28\frac{19}{35} = 28$$

Numbers divisible by 5 or 7 = 143 + 200 – 29 = 314

Hence, total numbers between (0 – 999) not divisible 5 or 7

= 1000 – 314 = 686

Q.2 If R and S are different integers both divisible by 5, then which of the following is not necessarily true?

(a) $R - S$ is divisible by 5

(b) $R + S$ is divisible by 10

(c) $R \times S$ is divisible by 25

(d) $R^2 + S^2$ is divisible by 5

[UPSC-2016]

Sol. **(b)**

If $R = 25$ and $S = 20$

then, $R + S = 25 + 20 = 45$

which is not divisible by 10

Hence (b) is not necessary true

Q.3 A 2-digit number is reversed. The larger of the two number is divided by the smaller one. What is the largest possible remainder?

(a) 9 (b) 27

(c) 36 (d) 45

[UPSC-2017]

Sol. **(d)**

For the largest possible remainder the number is = 49

The new number after reversing the digits = 94

$$49\overline{)94}\ \ 1$$
$$\underline{49}$$
$$45$$

thus the remainder = 45

Option (d) is correct

Q.4 There are certain 2-digits numbers. The difference between the number and the one obtained on reversing it is always 27. How many such maximum two digit numbers are there?

(a) 3 (b) 4

(c) 5 (d) None of the above

[UPSC-2017]

Sol. **(d)**

There are only 4 such 2 digits numbers

14, reversed number = 41, difference 41 – 14 = 27

25, reversed number = 52, difference 52 – 25 = 27

36, reversed number = 63, difference 63 – 36 = 27

47, reversed number = 74, difference 74 – 47 = 27

58, reversed number = 85, difference 85 – 58 =27

69, reversed number = 96, difference 96 – 69 =27

Option (d) is correct

Q.5 While writing all the numbers from 700 to 1000, how many numbers occur in which the digit at hundred's place is greater than the digit at ten's place, and the digit at ten's place is greater than the digit at unit's place?

(a) 61 (b) 64

(c) 85 (d) 91

[UPSC-2018]

Sol. **(c)**

700 – 1000

700 series

710	720	730	.....	760
1	2	3	.....	6

800 series

810	820	.....	870
1	2	.....	7

900 series

910	920	.....	980
1	2	.....	8

Total sum

= (1 + 2 + 3 ..... 6) + (1 + 2 ..... 7) + (1 + 2 ..... 8) = 85

Q.6 In a school every student is assigned a unique identification number. A student is a football player if and only if the identification number is divisible by 4, whereas a student is a cricketer if and only if the identification number is divisible by 6. If every number from 1 to 100 is assigned to a student, then how many of them play cricket as well as football?

(a) 4 (b) 8

(c) 10 (d) 12

[UPSC-2019]

Sol. **(b)**

Students playing both the sports = students with id 12(LCM of 4 and 6)

Numbers which are divisible by 12 between 1 – 100 = 8

Hence (b)

Q.7 An 8-digit number 4252746B leaves remainder 0 when divided by 3. How many values of B are possible?

(a) 2 (b) 3
(c) 4 (d) 6

[UPSC-2019]

Sol. **(c)**

4252746B is divisible by 3 so

4 + 2 + 5 + 2 + 7 + 4 + 6 + B = 3n

30 + B = 3n

B must be a multiple of 3

B = 0, 3, 6, 9

Hence (c)

Q.8 Number 136 is added to 5B7 and the sum obtained is 7A3, where A and B are integers. It is given that 7A3 is exactly divisible by 3. The only possible value of B is

(a) 2 (b) 5
(c) 7 (d) 8

[UPSC-2019]

Sol. **(d)**

7 + 6 = 13

That means 1 carries forward

So, B + 3 + 1 = 10 + A(as again 1 carries forward)

Therefore, B = A + 6

As 7A3 is divisible by 3, A can be 2, 5, 8

For A = 5, 8, B > 10 which is not possible so only possible value of A is 2

So, B = 8

Hence (d)

Q.9 Let XYZ be a three-digit number, where (X + Y + Z) is not a multiple of 3. Then (XYZ + YZX + ZXY) is not divisible by

(a) 3 (b) 9
(c) 37 (d) (X + Y + Z)

[UPSC-2020]

Sol. **(b)**

X + Y + Z is not equal to 3n where n is any natural number

XYZ + YZX + ZXY = 100(X + Y + Z) + 10(X + Y + Z) + (X + Y + Z)

= 111(X + Y + Z)

This is divisible by 3, 37, X + Y + Z

But not by 9

Hence (b)

Q.10 How many five-digit prime numbers can be obtained by using all the digits 1, 2, 4 3, 4 and 5 without repetition of digits?

(a) Zero (b) One
(c) Nine (d) Ten

[UPSC-2020]

Sol. **(a)**

Sum of 1, 2, 3, 4, 5 = 15

i.e. it will always be divisible by 3

So no such number possible

Hence (a)

Q.11 How many integers are there between 1 and 100 which have 4 as- a digit but are not divisible by 4?

(a) 5 (b) 11
(c) 12 (d) 13

[UPSC-2020]

Sol. **(c)**

All such numbers are:

14,34,41,42,43,45,46,47,49,54,74,94

i.e. 12 such numbers

Hence (c)

Q.12 What is the remainder when 51 × 27 × 35 × 62 × 75 is divided by 100?

(a) 50 (b) 25
(c) 5 (d) 1

[UPSC-2020]

Sol. **(a)**

The unit place of the expression will be zero

So the only possible option is 50

Hence (a)

Q.13 For what value of n, the sum of digits in the number $(10^n + 1)$ is 2?

(a) For n = 0 only
(b) For any whole number n
(c) For any positive integer n only
(d) For any real number n

[UPSC-2020]

Sol. **(b)**

10^n for any whole number n will have sum of digits as 1

Hence (b)

Q.14 A digit n > 3 is divisible by 3 but not divisible by 6. Which one of the following is divisible by 4?

(a) 2n (b) 3n
(c) 2n + 4 (d) 3n + 1

[UPSC-2020]

Sol. **(d)**

n= 9 is divisible by 3 but not 6

Option (d) satisfies the condition

As 3n + 1 = 28 is divisible by 4

Hence (d)

Q.15 What is the largest number among the following?

(a) $\left(\frac{1}{2}\right)^{-6}$ (b) $\left(\frac{1}{4}\right)^{-3}$

(c) $\left(\frac{1}{3}\right)^{-4}$ (d) $\left(\frac{1}{6}\right)^{-2}$

[UPSC-2020]

Sol. **(c)**

Option (a) : $\left(\frac{1}{2}\right)^{-6} = 2^6 = 64$

Option (b) : $\left(\frac{1}{4}\right)^{-3} = 4^3 = 64$

Option (c) : $\left(\frac{1}{3}\right)^{-4} = 3^4 = 81$

Option (d) : $\left(\frac{1}{6}\right)^{-2} = 6^2 = 36$

Q.16 Consider all 3-digit numbers (without repetition of digits) obtained using three non-zero digits which are multiples of 3. Let S be their sum. Which of the following is/are correct?

1. S is always divisible by 74

2. S is always divisible by 9

Select the correct answer using code given below:

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

[UPSC-2021]

Sol. **(c)**

Three non-zero digits which are multiples of 3 are: 3, 6, and 9

Using these 3 digits, we can make 6 three-digit numbers : 396, 369, 693, 639, 936, 963

So their sum = 369 + 396 + 639 + 693 + 936 + 963 = 3996

It is divisible by both 74 and 9

Hence (c)

Q.17 Integers are listed from 700 to 1000. In how many integers is the sum of the digits 10?

(a) 6 (b) 7
(c) 8 (d) 9

[UPSC-2021]

Sol. **(d)**

Here we have to find out all the integers between 700 to 1000, in which sum of the digits is 10,

Between 700 – 799

7 + a + b = 10

a + b = 3

possible numbers = 703, 712, 721 , 730

Between 800 – 899

8 + a + b = 10

a + b = 2

possible numbers = 802, 811, 820

Between 900 – 999

9 + a + b = 10

a + b = 1

possible numbers = 901, 910

total numbers = 10

Hence (d)

Q.18 If 3^{2019} is divided by 10, then what is the remainder?

(a) 1 (b) 3
(c) 7 (d) 9

[UPSC-2021]

Sol. **(c)**

In 3^{2019}, the unit place will be 7

So remainder will be 7

Hence (c)

Q.19 The number 3798125P369 is divisible by 7. What is the value of the digit P?

(a) 1 (b) 6
(c) 7 (d) 9

[UPSC-2021]

Sol. (d)

Using hit and trial method,

For the number to be divisible by 7,

P must be 6

Hence (d)

Q.20 A biology class at high school predicted that a local population of animals will double in size every 12 years. The population at the beginning of the year 2021 was estimated to be 50 animals. If P represents the population after n years, then which one of the following equations represents the model of the class for the population?

(a) P = 12 + 50n (b) P = 50 + 12n

(c) P = 50 (2)12n (d) P = 50 (2)n/12

[UPSC-2021]

Sol. (d)

Population is getting doubled every 12 years, and population in the year 2021 is 50 animals. So, after 12 years it will get doubled to 100 animals.

Using options for n = 12, in option d, P = 100

Hence(d)

Q.21 When a certain number is multiplied by 7, the product entirely comprises ones only (1111...). What is the smallest such number?

(a) 15713 (b) 15723

(c) 15783 (d) 5873

[UPSC-2021]

Sol. (d)

1 is not divisible by 7

11 is not divisible by 7

111 is not divisible by 7

1111 is not divisible by 7

11111 is not divisible by 7

111111 is divisible by 7 = 111111/7 = 15873

Hence (d)

Q.22 $15 \times 14 \times 13 \times \ldots \times 3 \times 2 \times 1 = 3^m \times n$

Where m and n are positive integers, then what is the maximum value of m?

(a) 7 (b) 6

(c) 5 (d) 4

[UPSC-2022]

Sol. (b)

$15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3^m \times n$

Numbers which are multiple of 3 = $15 \times 12 \times 9 \times 6 \times 3 = (3 \times 5) \times (3 \times 4) \times (3 \times 3) \times (3 \times 2) \times 3 = 3^6 \times (5 \times 4 \times 2)$

Therefore, the maximum value of m is 6

Hence, option (b) is the correct answer

Q.23 Which number amongst 2^{40}, 3^{21}, 4^{18} and 8^{12} is the smallest?

(a) 2^{40} (b) 3^{21}

(c) 4^{18} (d) 8^{12}

[UPSC-2022]

Sol. (b)

The given numbers are: 2^{40}, 3^{21}, 4^{18} and 8^{12}

We can also write them as: 2^{40}, 321, 2^{36} and 2^{36}

Q.24 What is the remainder when $91 \times 92 \times 93 \times 94 \times \times 95 \times 96 \times 97 \times 98 \times 99$ is divided by 1261?

(a) 3 (b) 2

(c) 1 (d) 0

[UPSC-2022]

Sol. (d)

Given expression = $91 \times 92 \times 93 \times 94 \times 95 \times 96 \times 97 \times 98 \times 99$

$1261 = 1 \times 13 \times 97$

So, its factors are 13 and 97

Since in expression X, multiples of 13 and 97 are there, 1261 will completely divide the expression X. Hence, the remainder = 0

PRACTICE SET : BASIC QUESTIONS

Q.1 What digit should be put in place of x in five-digit number 4398x to make it divisible by 3?

(a) 0 (b) 3

(c) 2 (d) Both (a) & (b)

Q.2 Find the unit digit of the expression

$1^2 + 2^2 + 3^2 + \ldots 50^2$

(a) 0 (b) 3

(c) 5 (d) 9

Q.3 Find the unit digit of the expression

$1^1 + 2^2 + 3^3 + 4^4 + 5^5 + \ldots + 10^{10}$

(a) 7 (b) 3
(c) 0 (d) None of these

Q.4 What is the remainder when $10 + 10^2 + 10^3 + 10^4$ is divided by 7 ?
(a) 1 (b) 3
(c) 5 (d) 2

Q.5 The remainder when $69^{69^{69^{\cdots}}}$ is divided by 34 is :
(a) 0 (b) 1
(c) 31 (d) Data Inadequate

Q.6 A 60 digit number is written using the positive integers in the form of 12345678910111213.... This number is divisible by ?
(a) 0 (b) 5
(c) 2 (d) 4

Q.7 The difference of $10^8 - 6$ and $10^7 - k$ is divisible by 3 for which value of k ?
(a) 0 (b) 1
(c) 2 (d) 3

Q.8 If the unit digit in the product ($39 \times 136 \times 48A \times 574$) is 4 the value of A is:
(a) 3 (b) 4
(c) 5 (d) 2

Q.9 If a number 774958P96Q is to be divisible by 8 and 9. The minimum value of $P + Q$ is :
(a) 16 (b) 9
(c) 8 (d) None of these

Q.10 Which of these is greatest?
(a) 2^{300} (b) 3^{200}
(c) 5^{150} (d) Either (a) or (b)

Q.11 Which of these is greater: 5^{100} or 2^{300}
(a) 5^{100} (b) 2^{300}
(c) Both are equal (d) Can't be determined.

Q.12 The value of $(\sqrt{8})^{\frac{1}{3}}$ is:
(a) 2 (b) $\sqrt{4}$
(c) $\sqrt{2}$ (d) 8

Q.13 If $\sqrt{2^n} = 64$, then the value of n is:
(a) 5 (b) 3
(c) 6 (d) 12

Q.14 If x = –0.5, then which of the following has the smallest value?
(a) $2\sqrt{-x}$ (b) $\frac{1}{x}$
(c) 2^{-x} (d) $\frac{1}{\sqrt{-x}}$

Q.15 If $(1.001)^{259} = 1.29$, $(1.001)^{62} = 1.06$, then $(1.001)^{321}$
(a) 1.14 (b) 1.8
(c) 2.23 (d) 1.37

Answer key

1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(a)
5.	(b)	**6.**	(b)	**7.**	(a)	**8.**	(b)
9.	(c)	**10.**	(c)	**11.**	(b)	**12.**	(c)
13.	(d)	**14.**	(b)	**15.**	(d)		

PRACTICE SET : ADVANCE QUESTIONS

Q.1 The single digit number that must be added to 108312 in order to obtain a multiple of 11 is:
(a) 2 (b) 8
(c) 6 (d) 5

Q.2 Find the two-digit number that meets the following criteria :
The digit in the unit's place exceeds the digit in its ten's by 2 and the product of the required number with the sum of its digits is equal to 144.
(a) 13 (b) 46
(c) 24 (d) 35

Q.3 A number when divided by 76 gives 53 as a remainder, find the remainder when this number is divided by 19.
(a) 9 (b) 14
(c) 16 (d) 15

Q.4 Which of the following can be a number divisible by 48 ?
(a) 513704 (b) 1387654
(c) 222144 (d) None of these

Q.5 If $x2387$ is divisible by 11, find the value of x?
(a) 5 (b) 1
(c) 7 (d) 0

Q.6 What is the largest possible two digit number by which 327635 can be divided?
(a) 55 (b) 65
(c) 85 (d) 95

Q.7 If 1872ab is divisible by 80. Then find the smallest value of (a + b):
(a) 4 (b) 0
(c) 8 (d) 6

Q.8 If 24753x is divisible by 36, then find the value of x:
(a) 0 (b) 4
(c) 6 (d) 8

Q.9 A number of the form $4x + 2$ is always divisible by 6, where x is a natural number, then:
(a) x is prime (b) x is even
(c) x is odd (d) None of these

Q.10 The smallest possible number which must be multiplied or divided to 10,000 to make it a perfect square is:
(a) 10 (b) 25
(c) 5 (d) 40

Q.11 The greatest divisor of
$(a^4 - b^4)(a^4 + b^4)(a^8 + b^8)(a^{16} + b^{16})(a^{32} + b^{32})$ is:
(a) $a^{16} - b^{16}$ (b) $a^{32} + b^{32}$
(c) $a^{64} - b^{64}$ (d) $a^{64} + b^{64}$

Q.12 $25^n - 1$ is:
(a) always divisible by 8
(b) always divisible by 3
(c) always divisible by 6
(d) All of these

Q.13 Which of the following can never be in the ending of a perfect cube?
(a) 6 (b) Only one '0'
(c) 4 (d) 5

Q.14 The square root of 528529 is:
(a) 413 (b) 727
(c) 517 (d) 567

Q.15 $\sqrt{729} \div \sqrt{a^4} = 3$, then the value of a is:
(a) 27 (b) 18
(c) 81 (d) 3

Q.16 Which one of these is smallest ?
$\sqrt[3]{5}$ or $\sqrt[4]{3}$ or $\sqrt[5]{4}$
(a) $\sqrt[3]{5}$
(b) $\sqrt[4]{3}$
(c) $\sqrt[5]{4}$
(d) Cannot be determined

Q.17 If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5p + 3$, then the value of $p + 6$ is:
(a) 9 (b) 12
(c) 3 (d) 4

Q.18 The value of $\dfrac{2-\sqrt{3}}{2+\sqrt{3}}$ is :
(a) $9-4\sqrt{5}$ (b) $9+4\sqrt{5}$
(c) $7-4\sqrt{3}$ (d) $7+4\sqrt{3}$

Q.19 The value of $\dfrac{1}{\sqrt{5}+\sqrt{3}} + \dfrac{1}{\sqrt{7}+\sqrt{5}}$ is:
(a) 1 (b) $\dfrac{\sqrt{7}-\sqrt{3}}{2}$
(c) $\sqrt{5}$ (d) $\dfrac{\sqrt{7}+\sqrt{3}}{2}$

Q.20 If $(\sqrt{a}+\sqrt{b}) = 13$ and $(\sqrt{a}-\sqrt{b}) = \sqrt{29}$, then the value of $\sqrt{ab}$ is :
(a) 50 (b) 40
(c) 160 (d) 35

Q.21 The value of $\left(\dfrac{a^n + a^{-n}}{2}\right)^2 - \left(\dfrac{a^n - a^{-n}}{2}\right)^2$ is:
(a) 0 (b) 1
(c) 4 (d) None of these

Q.22 $(2^{13} + 1)$ is divisible by:
(a) 2 (b) 3
(c) 5 (d) None of these

Q.23 $2^{41} - 2^{39} - 2^{40}$ is same as:

(a) 2^{41} (b) 2^{39}

(c) 2^{37} (d) 2^{40}

Q.24 If $3^{a-1} + 3^{a} + 3^{a+1} = 1053$, then the value of a is:

(a) 1 (b) 2

(c) 5 (d) 4

Q.25 If $x = \sqrt{3} + \sqrt{2}$, then the value of $x^3 + x + \frac{1}{x} + \frac{1}{x^3}$ is

(a) $15\sqrt{3}$ (b) $20\sqrt{3}$

(c) $10\sqrt{2}$ (d) $15\sqrt{2}$

Q.26 Which one of the following is correct?

(a) $\sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}$ (b) $\sqrt{2} < \sqrt[4]{6} > \sqrt[3]{4}$

(c) $\sqrt[4]{6} < \sqrt{2} < \sqrt[3]{4}$ (d) $\sqrt[4]{6} > \sqrt{2} < \sqrt[3]{4}$

Answer key

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(c)
5.	(d)	**6.**	(a)	**7.**	(b)	**8.**	(c)
9.	(d)	**10.**	(b)	**11.**	(c)	**12.**	(d)
13.	(b)	**14.**	(b)	**15.**	(d)	**16.**	(b)
17.	(a)	**18.**	(c)	**19.**	(b)	**20.**	(d)
21.	(b)	**22.**	(b)	**23.**	(b)	**24.**	(c)
25.	(b)	**26.**	(a)				

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